This generalizes the usual fixed end condition u = 0. The second is

$$\partial^{2s-1}u/\partial x^{2s-1} = 0, s = 1, 2, 3, \ldots \tag{26}$$

This generalizes the usual free fixed end condition $N = B_1(\partial u/\partial x) = 0$. These will be referred to in this paper as the fixed and free end conditions respectively.

Three cases are considered. They are a lattice fixed at both ends, a lattice free at both ends, and a lattice fixed at the left end and free at the right end. (In all cases the left end is at x = 0 and the right end is at x = L.) In each case the solution can be written as a super-position of normal modes in the form

$$u(x,t) = \sum_{n=1}^{\infty} F_n(x)G_n(t) \tag{27}$$

Separation of variables leads to the solution for the modal amplitudes

$$G_n(t) = A_n \sin(\omega_n t) + B_n \cos(\omega_n t) \tag{28}$$

where ω_n is the natural frequency of the nth normal mode and A_n and B_n are constants. For the lattice fixed at both ends

$$F_n(x) = \sin(n\pi x/L) \tag{29}$$

and

$$\omega_n^2 = (1/\rho) \sum_{s=1}^{\infty} (-1)^{s-1} (n\pi/L)^{2s} B_s = (4/m) \sum_{q=1}^{p} k_q \sin^2(nq\pi d/(2L)) \tag{30}$$

For a chain that is free at both ends

$$F_n(x) = \cos(n\pi x/L) \tag{31}$$

and the natural frequencies are again given by (30). In this case a rigid-body mode must be added to (27) to obtain the most general solution. For a lattice with one fixed and one free end

$$F_n(x) = \sin((2n-1)\pi x/(2L)) \tag{32}$$

and

$$\omega_n^2 = (1/\rho) \sum_{s=1}^{\infty} (-1)^{s-1} ((2n-1)\pi x/(2L))^{2s} B_s = (4/m) \sum_{q=1}^{p} k_q \sin^2((2n-1)\pi x/(4L)) \tag{33}$$

By comparison with the result of Chen (1971) who for the case of nearest neighbor interactions only obtained results corresponding to those of this section by direct solution of (1), it was found that for a chain of r masses the length L contained in the above equations is found by adding d/2 for each fixed and present to the product rd. Thus for the fixed-fixed case L = (r+1)d, for the free-free case L = rd, and for the fixed-free case L = (r + (1/2))d.

The natural frequencies that would be found using an approximate continuum theory obtained by keeping only the first j terms in the series expansion in (7) can be determined by expanding the trigonometric functions in (30) and (33) and retaining the first j terms. The first few terms of such an expansion will be accurate only if the arguments of the trigonometric functions are small. Thus it can be seen that this type of approximate theory will yield solutions that are more accurate for low modes than for high ones, more accurate for long chains than for short ones (the chain is long if (d/L) << 1), and more accurate for chains where long-range interactions are absent than for those where they are present. These conclusions are quite similar to those drawn from an examination of the problem of propagation of harmonic waves in an infinite lattice. The problems discussed in this section were also solved for the other two lattices previously discussed. The effects of non-local elasticity were found to be the same.

CONCLUSIONS

In the present paper continuum descriptions of three one-dimensional lattices have been developed. It was found that the resulting equations corresponded to theories of elasticity which take long-range elastic effects into consideration. For the case of a chain of identical mass points exact solutions to the equations were found for wave propagation and vibration problems. These solutions were used to assess the importance of long-range elastic effects and the accuracy of various approximations to the exact continuum equations.

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FRAGRANCE ANALYSES OF *TRILLIUM LUTEUM AND TRILLIUM CUNEATUM* (Liliaceae)

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ABSTRACT

The flower fragrances of *Trillium cuneatum* and *T. luteum* were analyzed by means of gas chromatography. A total of 12 different compounds were detected, only two of which occurred in both taxa. Four compounds were specific to *T. luteum*, six to *T. cuneatum*. The identity of only one compound, the terpene alcohol linalool, was determined. This is the major fragrance component of *T. luteum* and is believed to be the compound responsible for the lemony odor so characteristic of the flowers of the plant. Taxonomic relationships of *T. luteum* and *T. cuneatum* are discussed.

INTRODUCTION

Despite the abundance, widespread distribution and relatively large, often showy flowers of species of *Trillium*, little is known concerning the nature of pollination and the species identity of pollen vectors. While the flowers of some of the species have been long known to emit a disagreeable odor and to attract flies, others produce an agreeable fragrance not obviously attractive to specific insects. The flowers of still others are apparently odorless. The odors reported include spicy, sweet, musky, rose, lemon, aminoid and foetid, but nowhere is information available on the chemistry of any of the compounds.

The present report is a chromatographic comparison of the fragrance compounds of the flowers of two closely related kinds of *Trillium* of eastern United States (subgenus Phyllantherum Raf.), the taxonomic treatment of which has been the subject of disagreement. The chemical data constitute information believed to be useful in investigations, not only taxonomic but biological, in the sense of providing a basis for and an impetus to future works along the lines of pollination ecology and hybridization.

MATERIALS AND METHODS

Plants of *Trillium luteum* and *T. cuneatum* were collected near Ozone, Cumberland County, and Nashville, Davidson County, Tennessee, respectively, in the early spring of 1969. The plants were transplanted to a greenhouse at the University of Miami where they soon flowered and flower fragrance analyses were performed by the first author. Following anthesis flower fragrances were obtained by one of the following two methods. By the first method intact scapes were individually enclosed within airtight plexiglass chambers (box-like in design) the outlets of which were equipped with swage lock fittings and self-sealing septa. This procedure minimized damage to the plants and permitted sampling to be performed throughout the flowering period. By the second method whole flowers were removed from the plants and several of these placed within airtight glass bottles, the tops of which were also equipped with swage lock fittings and self-sealing septa. Although the latter procedure did not afford the advantages of repeated sampling over a period of several days, it did allow sampling over a period of several hours and permitted a comparatively more concentrated sample to be obtained.

The chambers were equilibrated for 30 minutes to allow the internal atmosphere to become as fully saturated with fragrance compounds as possible. Samples of the fragrance-containing atmosphere were extracted in 10 cc volumes with the use of a gas-tight syringe.

Analyses were performed isothermally at oven temperatures of 70°, 100°, 130° and 160°C with an F & M model 810 dual flame gas chromatograph equipped with a 1:1 effluent splitter. At each temperature a 10cc test sample was injected into each of two different 6 ft. x ¼ in. stainless steel columns. One of the columns was packed with 3% carbowax 20M on 80-100 mesh Diatoport S, the other with 10% Lac 446 (diethylene glycol adipate) on 80-100 mesh Chromosorb W. The effluent splitter permitted the compounds to be examined olfactorally as they eluted from the column. However, the extremely small quantity of each compound in the injected air sample precluded its collection for use in other analytical procedures. Quantitative measurements were obtained for each sample from an integrator trace and expressed as a percentage of the injected sample. Fragrance components were identified by calculation of relative retention times of the peaks using the standards: β-pinene at 70°, 2-phenylethylacetate at 100°, ethyl benzoate at 130°, and 2-phenylethanol at 160°C.

RESULTS

Peaks obtained on the tracings were assigned arbitrary numbers on the basis of increasing relative retention times and increasing temperatures. Certain peaks with high relative retention times at 70° as well as those with low values at 130°C could also be detected at 100°C. Such peaks were assumed to be identical on the basis of similar size, similar elution odor, and projected decrease or increase in relative retention time accompanying the specific temperature increase or decrease, respectively. Equivalence of peaks on the two types of columns was judged on the basis of the same criteria. Repeated analyses using the techniques described indicated that the floral fragrances of *Trillium luteum* and *T. cuneatum* consist of a total of 12 different compounds. The relative retention time and percentage composition of each compound are given in Table 1.

The floral fragrance of *Trillium luteum* was found to consist of six different chemical compounds. Of the six compounds detected, peak 11 was the major component, accounting for slightly over 70 percent of the total fragrance. This was also the only compound that had a detectable odor as it eluted from the column. This odor was distinctly lemon-like. Comparison of the relative retention times of the fragrance compounds of *Trillium luteum* against retention times of the standards used suggested that peak 11 was probably the terpene

Table 1: Relative retention time and percentage composition of chemical compounds in injected samples of the floral fragrances of *Trillium luteum* and *Trillium cuneatum*.

Temp.	Peak No.	Relative Retention Time Carbowax	Lac 466	% Composition in *T. luteum*	*T. cuneatum*
70°	1	0.42	0.50	3.2	
	2	0.48	0.57		11.6
	3	0.65	0.63	9.4	36.3
	4	0.85	--		3.7
	5	1.01	1.00		10.6
	6	1.33	1.31	1.9	4.6
	7	1.61	1.82		6.7
	8	2.23	--	2.4	
100°	6	0.35	--		
	8	0.51	--		
	9	1.36	1.14		
	11	2.08	1.79		
130°	9	0.40	0.38	12.2	
	10	0.51	--		12.4
	11	0.55	0.52	70.9	
	12	0.75	--		14.1

Note: Peaks 6 and 8 were detected at both 70° and 100°; peaks 9 and 11 at 100° and 130°C.

alcohol linalool. This possibility was checked by augmentation. For this purpose head space samples of standard linalool were run on both columns at 130°C. The relative retention times and odor of linalool as it eluted from the column were found to be identical to that of peak 11. By augmentation a sample of the flower fragrance was enriched with a small amount of standard linalool (head space sample) and this mixture analyzed. An increase in the size of the peak was obtained. This result, without appearance of a shoulder, was taken as a good indication that linalool was the major fragrance component. As a further check, the temperature was dropped to the next lowest standard level (100°C) and the augmentation procedures repeated. (cf., Table 1). The fragrance compound and the standard linalool were found to have identical relative retention times on both columns, and enrichment again produced a single enlarged peak without a shoulder.

It was concluded from these data that linalool is the major fragrance compound produced by the flowers of *Trillium luteum,* and is the compound responsible for the lemony fragrance which the flowers produce. The other compounds produced by the flowers probably alter slightly the odor of linalool inasmuch as the fragrance produced by the flowers smells slightly sweeter than the odor of pure linalool.

The floral fragrance of *Trillium cuneatum* was found to consist of eight compounds. The major compound, peak 3, accounted for approximately 36 percent of the total fragrance. This peak did not, however, have a detectable odor as it eluted from the column, nor did any of the other compounds that were present. Of the eight compounds detected, only one possessed a relative retention time similar to that of any available standard compound. Peak 5 had a relative retention time very similar to that of β-pinene. It was not possible to equate peak 5 with β-pinene because the standard β-pinene had a very distinct and easily detected odor as it eluted from the chromatographic column even when used in quantities comparable to those of peak 5.

The fragrance profiles of the two species were quite different. Only two compounds, peaks 3 and 6, were found to occur in both, but the quantities of each of the two compounds were noticeably different. Both peaks 3 and 6 occurred in *Trillium cuneatum* in quantities about three times greater than those quantities found to occur in *T. luteum.*

DISCUSSION

Trillium luteum (Huhl.) Harb. is distributed in the Ridge and Valley and southern Appalachian provinces of eastern Tennessee, Kentucky, western North Carolina and northwestern Georgia. The flowers are yellowish green to bright yellow, relatively uniform in color and emit a pleasant lemon-like fragrance. The geographical range of *T. luteum* (Fig. 1) is essentially surrounded, except to the northeast, by that of the comparatively widespread and closely related *T. cuneatum* Raf.

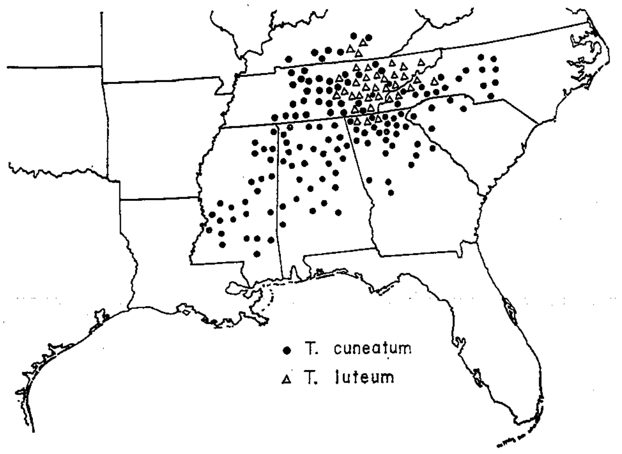

Fig. 1: Distribution of *Trillium luteum* and *T. cuneatum.*

Trillium cuneatum (excluding *T. luteum* consists of highly variable populations of individuals distributed geographically (Fig. 1) over the Blue Ridge and Piedmont provinces, ranging from the Carolinas southwestward across Georgia, Alabama and Mississippi and northward through Tennessee into the Highland Rim of Kentucky. The flowers are brown-purple or maroon, sometimes greenish, greenish yellow, rarely yellow, highly variable in color, and emit a sweet spicy fragrance. Plants with greenish yellow flowers are in the minority in this species, and individuals with yellow flowers are rare indeed. There is some evidence that flowers maroon in early anthesis often become greenish in age.

The close morphological similarities of these two plants have naturally led to disagreement not only as to identification, but to taxonomic treatment as well. *Trillium luteum* has been variously treated as a species and as a geographical variety of *T. cuneatum.*

While differences in flower fragrances are useful in distinguishing *Trillium luteum* and *T. cuneatum,* they may also provide clues to possible hybridization between these taxa. Freeman (1969), in his taxonomic treatment of sessile-flowered species of *Trillium,* has examined populations including putative hybrids of these taxa in Pulaski and Russell counties of south-central Kentucky (*Freeman 473, 477, 479,* VDB). In this area the Cumberland Plateau, which spacially isolates *T. cuneatum* in Middle Tennessee from *T. luteum* at its westernmost distributional limits, has been dissected by the Cumberland River, and the ranges of the taxa here come together. The plants in these populations bear mostly yellow colored, lemon scented flowers, although plants bearing purple colored, spice-scented flowers also occur. In addition to the typical plants, occasional individuals are encountered the flowers of which are purple and produce a strong lemon odor.

Serota (1969), in connection with a study of morphological and karyotypic variation among sessile-flowered species of *Trillium* represented in North Carolina, commented on the dependability of fragrance differences between *T. luteum* and *T. cuneatum.* She reported observations of a population near Fontana Dam in Polk County containing putative hybrids between the two taxa. She observed plants with yellow flowers having either a definite spice scent or, more frequently, without any fragrance at all, but she was unable to detect a lemon fragrance in any flowers bearing visible traces of anthocyanin. Serota concluded that the morphological and karyotypic similarities of *T. luteum* and *T. cuneatum* warrant consideration of the two as a single taxonomic species.

Freeman (loc. cit.) discussed the apparent close relationships of the two taxa but concluded, correctly in our opinion, that both must be accorded taxonomic status above that of formae of a single species because of their largely allopatric patterns of distribution. He chose, perhaps somewhat arbitrarily, but not without careful and searching deliberation, to recognize each in species status. This was done so as not to disturb unduly or prematurely the system of hierarchy and the nomenclature of the component elements judged to be the most reasonable for the subgenus as a whole on the basis of currently available information. It is clear that convincing arguments may even now be leveled to justify the treatment of *T. luteum* as a geographical variety or as a subspecies of *T. cuneatum,* but there seems to be insufficient biological evidence at the present time to warrant such manipulation. Indeed, there are obvious practical advantages in retaining *T. luteum* as a taxonomic species.

The results of the present study of flower fragrances do not settle the question of the taxonomic rank of these two plants. Although 12 different fragrance compounds were detected in the two, only two of the compounds occurred in the flowers of both taxa where they exhibited approximately a 3-fold quantitative difference. If these data alone were used as an indication of relationship, the two taxa would appear to be quite different.

But when data relative to the occurrence of flavonoids and other flourescent compounds in the flowers of these plants are compared, (Murrell, 1969), considerable similarity is seen to exist. *Trillium cuneatum* was found to contain some 44 chromatographically recognizable fluorscent components while *T. luteum* contained a total of 43 such compounds. Of this group of compounds 36 were common to both. The possibility of different taxonomic interpretations arising from the separate evaluation of the two types of chemical data emphasizes the importance of a synthesizing approach to the final solution of this taxonomic problem, using information from all possible sources.

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BIOMASS AND PRODUCTIVITY ESTIMATES FOR A TEMPERATE MESIC SLOPE FOREST

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ABSTRACT

Aboveground biomass (420 t/ha) and net annual aboveground productivity (1510 $g/m^2/yr$) of a mature mixed deciduous forest were estimated from empirically determined stem diameter measurements on 360 canopy (⩾10.16 cm dbh) trees and published stem diameter: biomass regression equations. The resulting estimates as well as the basal area coverage (43 m^2/ha) lay within limits encountered in temperate deciduous forests

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